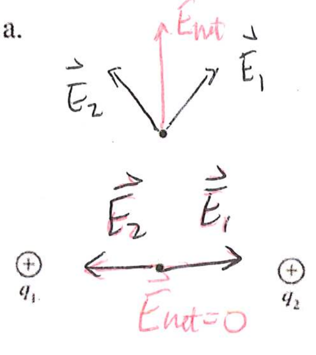
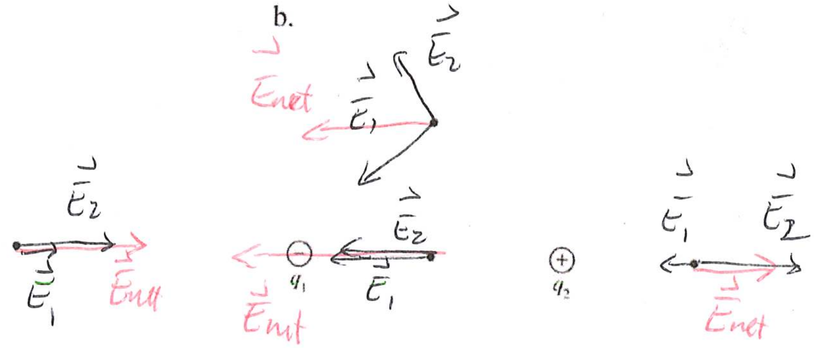
1.

At each of the dots, use a **black** pen or pencil to draw and label the electric fields $\vec{E}_1$ and $\vec{E}_2$ due to the two point charges. Make sure that the *relative* lengths of your vectors indicate the strength of each electric field. Then use a **red** pen or pencil to draw and label the net electric field $\vec{E}_{net}$ at each dot.

a.



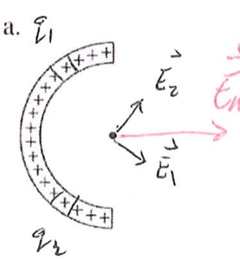
b.



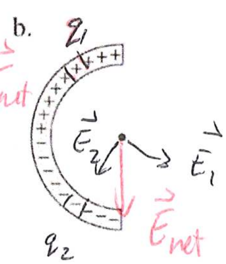
2.

The figure shows two charged rods bent into a semicircle. For each, draw the electric field vector at the dot at the "center" of the semicircle.

a.



b.

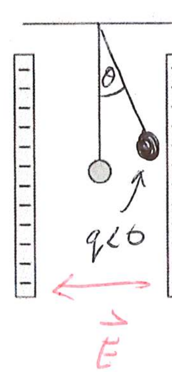


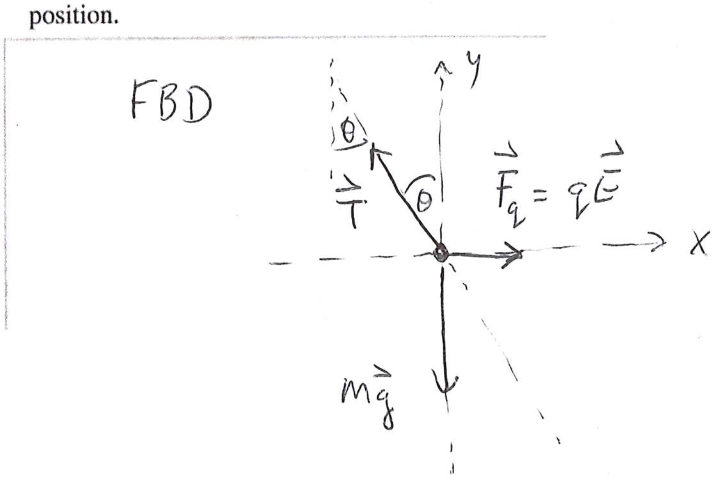
3.

A ball hangs from a thread between two vertical capacitor plates. Initially, the ball hangs straight down. The capacitor plates are charged as shown, then the ball is given a small negative charge. The ball moves to one side, but not enough to touch a capacitor plate.

a. Draw the ball and thread in the ball's new equilibrium position.

b. In the space below, draw a free-body diagram of the ball when in its new position.



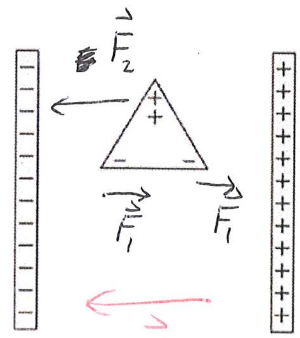


4. Three charges are placed at the corners of a triangle. The ++ charge has twice the quantity of charge of the two − charges; the net charge is zero.

a. Draw the force vectors on each of the charges.

b. Is the triangle in equilibrium? No If not, draw the equilibrium orientation directly beneath the triangle that is shown.

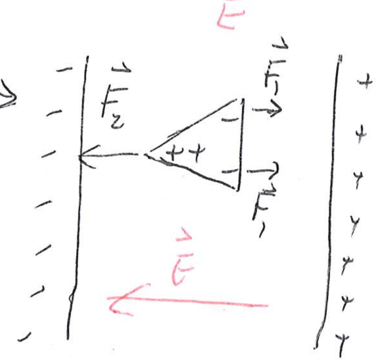
c. Once in the equilibrium orientation, will the triangle move to the right, move to the left, rotate steadily, or be at rest? Explain.



Forces tend to cause triangle to rotate

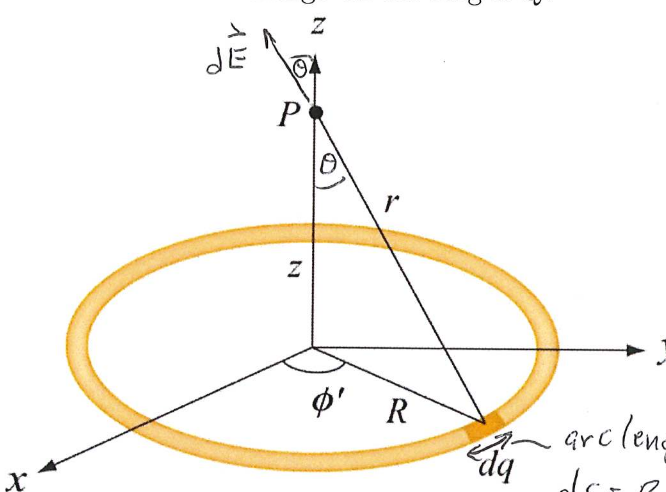
In the equilibrium position, $\vec{F}_2 + 2\vec{F}_1 = 0$ & there is no additional rotation.

∴ triangle is at rest.



If you managed to get through the previous problems quickly, here are a couple of more challenging problems... If you get stuck, follow the derivations given in section 23.4 of the textbook.

5. (a) Find the electric field due to a uniformly charged ring of radius R at point P in the figure below. The ring is in the x-y plane with its centre at the origin and point P is on the z-axis. Assume that the total charge on the ring is Q.



arc length $ds = R\,d\phi'$

dE due to dq is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

$$r^2 = z^2 + R^2$$

$$\lambda = \frac{Q}{2\pi R} \qquad dq = \lambda\, ds = \lambda R\, d\phi'$$

continued on back.

(b) Confirm that the electric field that you calculated in (a) looks like that of a point charge when $z \gg R$.

We need only the z-component of $d\vec{E}$ (components parallel to xy-plane will cancel.

$$dE_z = dE\cos\theta \qquad \cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2+R^2}}$$

$$\therefore dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\,d\phi'}{z^2+R^2}\frac{z}{\sqrt{z^2+R^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda R z}{(z^2+R^2)^{3/2}}\,d\phi'$$

$$E_z = \int_{\phi'=0}^{2\pi} dE_z = \int_{\phi'=0}^{2\pi}\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{\lambda R z}{(z^2+R^2)^{3/2}}}_{\text{constants.}}\,d\phi'$$

$$\therefore E_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R z}{(z^2+R^2)^{3/2}}\underbrace{\int_{\phi'=0}^{2\pi} d\phi'}_{2\pi}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\overbrace{(2\pi R\lambda)}^{Q}\,z}{(z^2+R^2)^{3/2}}$$

$$\therefore\ \boxed{E_z = \frac{Q}{4\pi\varepsilon_0} \frac{z}{(z^2+R^2)^{3/2}}}$$

or $$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{z}{(z^2+R^2)^{3/2}}\ \hat{k}}$$

(b) when $z \gg R$

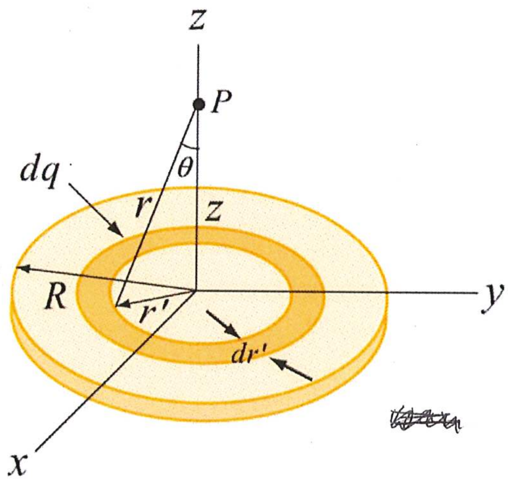
$$(z^2+R^2)^{3/2} \approx (z^2)^{3/2} = z^3$$

$$\therefore\ \vec{E} \approx \frac{Q}{4\pi\varepsilon_0} \frac{z}{z^3}\ \hat{k}$$

$$= \frac{Q}{4\pi\varepsilon_0 z^2}\ \hat{k}$$

like a point charge w/ P a dist. z from Q on z-axis.

6. (a) Find the electric field due to a uniformly charged disk of radius R at point P in the figure below. The disk is in the x-y plane with its centre at the origin and point P is on the z-axis. Assume that the total charge on the disk is Q. Do this problem by adding up the electric fields due to a series of rings that combine to form the disk.



charge of ring is $Q_i = \eta 2\pi r' dr'$

$\therefore$ E_i at P is

$$E_i = \frac{\eta 2\pi r' dr' \; z}{4\pi \varepsilon_0 \left(z^2 + r'^2\right)^{3/2}}$$

$$= \frac{\eta z}{2\varepsilon_0} \frac{r' dr'}{\left(z^2 + (r')^2\right)^{3/2}}$$

Specifically, show that the electric field at P can be expressed as:

$$E = \frac{\eta z}{2\varepsilon_0} \int_{r'=0}^{R} \frac{r'\, dr'}{\left(z^2 + (r')^2\right)^{3/2}}$$

where η is the charge density (i.e. charge per unit area) of the disk. The integral evaluates to:

$$\int_{r'=0}^{R} \frac{r'\, dr'}{\left(z^2 + (r')^2\right)^{3/2}} = \frac{1}{z} - \frac{1}{\sqrt{z^2 + R^2}}.$$

Therefore, the electric field at P due to the charged disk is:

$$E = \frac{\eta}{2\varepsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right].$$

(b) Confirm that the electric field that you calculated in (a) looks like that due to an infinite plane of charge density η when $R \gg z$.

$$E_{net} = \sum_i E_i = \frac{\eta z}{2\varepsilon_0} \sum_i \frac{r_i' dr'}{(z^2 + (r_i')^2)^{3/2}}$$

In limit $dr' \to 0$

$$\boxed{E_{net} = \frac{\eta z}{2\varepsilon_0} \underbrace{\int_0^R \frac{r' dr'}{(z^2 + (r')^2)^{3/2}}}_{\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}}}$$

look up in a table.

$$\therefore E_{net} = \frac{\eta z}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right] = \boxed{\frac{\eta}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]}$$

(b) If $R \gg z$, then $\frac{z}{\sqrt{z^2+R^2}} \approx \frac{z}{R} \to 0$

$$\boxed{\therefore E_{net} \approx \frac{\eta}{2\varepsilon_0}}$$

Infinite plane of charge.